BASIC GRAPHS

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A subclass of perfect graphs called basic graphs plays an essential role in the announced proof of the Strong Perfect Graph Conjecture by Chudnovsky, Robertson, Seymour and Thomas [1]. Recall that the *line graph* $L(G)$ of a graph G is the intersection graph of edges of G, that is $V(L(G)) = E(G)$ and two distinct vertices e and e' are adjacent in $L(G)$ if and only if the edges e and e' of G have a common vertex.

The following notation is used: $\mathcal{B}$ is the class of all bipartite graphs, $\overline{\mathcal{B}}$ is the class of complements of all bipartite graphs, $\mathcal{LB}$ is the class of line graphs of all bipartite graphs, and $\overline{\mathcal{LB}}$ is the class of complements of line graphs of all bipartite graphs.

Definition (Conforti, Cornuéjols and Vušković [2]). *The class of* basic graphs *is defined as*

$$\mathcal{BASIC} = \mathcal{B} \cup \overline{\mathcal{B}} \cup \mathcal{LB} \cup \overline{\mathcal{LB}}.$$

The well-known König theorem states that a graph is bipartite if and only if it does not contain any odd cycles as an induced subgraph. It follows that a Berge graph is bipartite if and only if it is C_3-free. Accordingly, a Berge graph is cobipartite if and only if it is O_3-free. Hemminger and Beineke [3] and independently Staton and Wingard [4] proved that the class $\mathcal{LB}$ coincides with the class of (Claw,Diamond,Odd Holes)-free graphs. Therefore, the class $\mathcal{LB}$ is exactly the class of (Claw, Diamond)-free Berge graphs, and $\overline{\mathcal{LB}}$ is exactly the class of (coClaw, coDiamond)-free Berge graphs.

The following theorem provides a characterization of the entire class of basic graphs in terms of forbidden induced subgraphs.

Theorem 1. *A graph G is basic if and only if it does not contain any of the graphs $G_1, G_2, \ldots, G_{16}$ (Figure 1), odd holes and odd antiholes as induced subgraphs.*

References

[1] M. Chudnovsky, N. Robertson, P. D. Seymour and R. Thomas, The Strong Perfect Graph Theorem, 2002, http://arxiv.org/PS_cache/math/pdf/0212/0212070.pdf

[2] M. Conforti, G. Cornuéjols and K. Vušković, Square-free perfect graphs, J. Combinatorial Theory, Ser.B (to appear)

[3] R. H. Hemminger and L. Beineke, Line graphs and line digraphs, in: *Selected Topics in Graph Theory* (Academic Press, 1978) 271–306.

[4] W. Staton and G. C. Wingard, On line graphs of bipartite graphs, Util. Math. **53** (1998) 183–187.

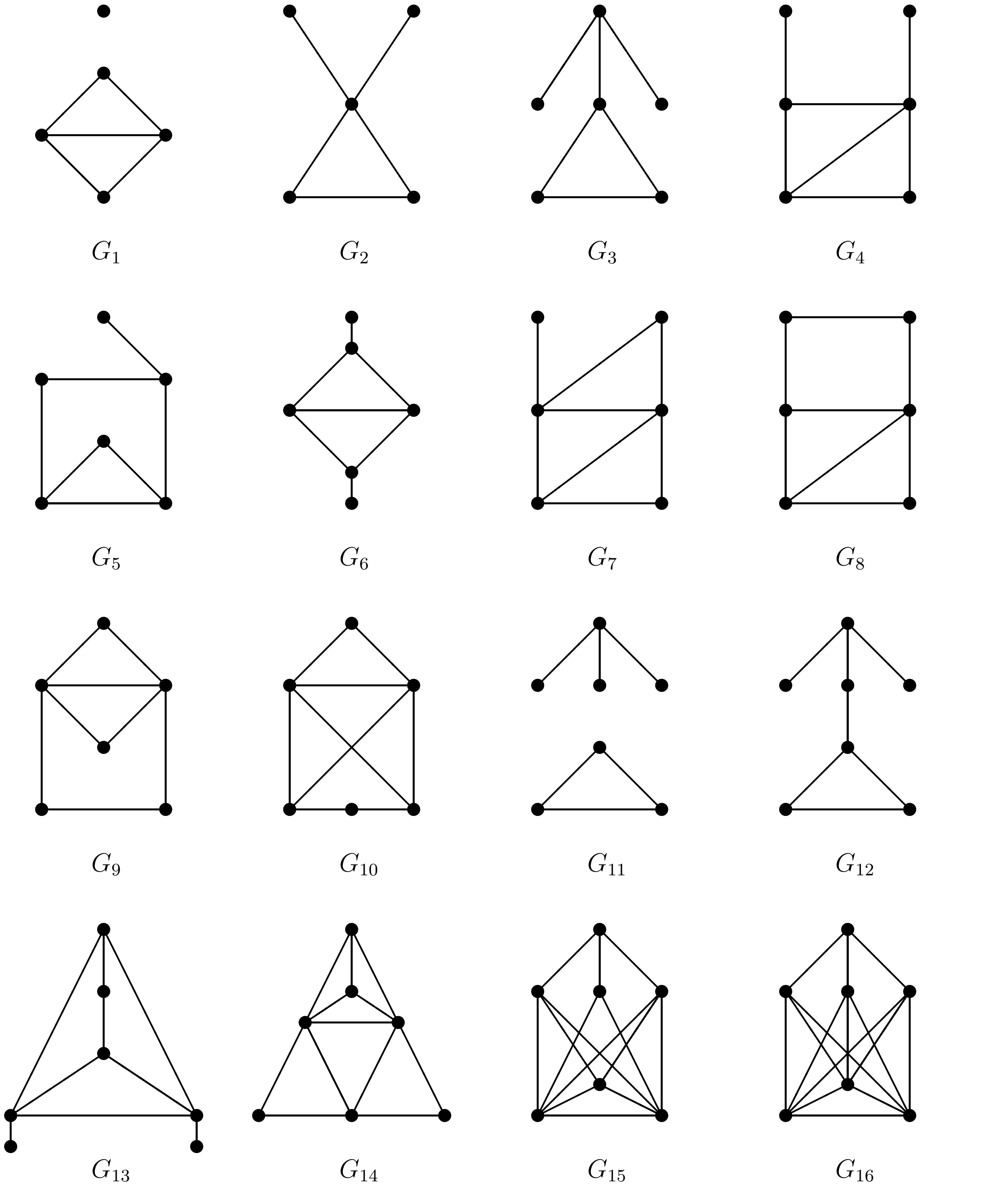

Figure 1. Forbidden induced subgraphs for basic graphs within Berge graphs.