A Disproof of Henning's Conjecture on Irredundance Perfect Graphs *

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Abstract

Let $ir(G)$ and $\gamma(G)$ be the irredundance number and the domination number of a graph G, respectively. A graph G is called *irredundance perfect* if $ir(H) = \gamma(H)$, for every induced subgraph H of G. In this paper we disprove the known conjecture of Henning [3, 11] that a graph G is irredundance perfect if and only if $ir(H) = \gamma(H)$ for every induced subgraph H of G with $ir(H) \le 4$. We also give a summary of known results on irredundance perfect graphs. Moreover, the irredundant set problem and the dominating set problem are shown to be NP-complete on some classes of graphs. A number of problems and conjectures are proposed.

Keywords: *irredundance perfect graphs; irredundance number; domination number*

1 Introduction

All graphs will be finite and undirected, without loops and multiple edges. If G is a graph, $V(G)$ denotes the set, and $|G|$ the number, of vertices in G. The edge set of G is denoted by $E(G)$. We write $u \perp X$ if the vertex u is adjacent to all vertices of the set $X \subseteq V(G)$, and $u \pm X$ if u is adjacent to no vertex of X. Let $N(x)$ denote the neighborhood of a vertex x, and let $\langle X \rangle$ denote the subgraph of G induced by $X \subseteq V(G)$. Also let $N(X) = \cup_{x \in X} N(x)$ and $N[X] = N(X) \cup X$.

A set $X \subseteq V(G)$ *dominates* a set $Y \subseteq V(G)$ if $Y \subseteq N[X]$. In particular, if X dominates $V(G)$, then X is called a *dominating set*. The *independent domination number* $i(G)$ is the cardinality of a minimum independent dominating set, and the *domination number* $\gamma(G)$ is the cardinality of a minimum dominating set of G. For $x \in X$, the set

$$N[x] - N[X - \{x\}]$$

*Supported by the Alexander von Humboldt Foundation, and the INTAS and the Belarus Government, Project INTAS-BELARUS 97-0093.

is called the *private neighborhood* of x and is denoted by $PN(x, X)$, or simply $PN(x)$ if X is clear from the context. If $PN(x, X) = \emptyset$, then x is said to be *redundant* in X. A set X containing no redundant vertex is called *irredundant.* The minimum cardinality taken over all maximal irredundant sets of G is the *irredundance number* $ir(G)$. A maximal irredundant set of cardinality $ir(G)$ is called an *ir-set.*

It is well known that for any graph G,

$$ir(G) \leq \gamma(G) \leq i(G).$$

Definition 1 *A graph G is called* domination perfect *if* $\gamma(H) = i(H)$*, for every induced subgraph H of G.*

Definition 2 *A graph G is called* irredundance perfect *if* $ir(H) = \gamma(H)$*, for every induced subgraph H of G.*

Definition 3 *A graph G is called* k-irredundance perfect $(k \geq 1)$ *if* $ir(H) = \gamma(H)$*, for every induced subgraph H of G with* $ir(H) \leq k$*.*

Denote by $\mathcal{IP}_k$ the class of k-irredundance perfect graphs. Since any graph is 1-irredundance perfect, $\mathcal{IP}_1$ is exactly the class of all graphs. Obviously,

$$\mathcal{IP}_1 \supseteq \mathcal{IP}_2 \supseteq \mathcal{IP}_3 \supseteq ...$$

Moreover, if $\mathcal{IP}$ is the class of irredundance perfect graphs, then

$$\mathcal{IP} = \bigcap_{i=1}^{\infty} \mathcal{IP}_i.$$

Definition 4 *A graph G is* minimal irredundance imperfect *if G is not irredundance perfect and* $ir(H) = \gamma(H)$*, for every proper induced subgraph H of G.*

There are a lot of interesting results on irredundance perfect graphs [2, 3, 7, 10, 11, 14, 15, 18], see Section 2 for a short summary. The problem of characterizing irredundance perfect graphs in terms forbidden induced subgraphs was posed by Henning [11] who noted that such a characterization is hard to obtain.

The related classes of graphs such as domination perfect graphs, upper domination perfect graphs and upper irredundance perfect graphs are studied as well. For a short survey on domination perfect graphs, see [21], and for a short survey on upper domination perfect graphs and upper irredundance perfect graphs, see [22]. While the irredundance and domination numbers are equal for irredundance perfect graphs, in general this is not the case. A number of authors [1, 2, 4, 8, 9, 12, 17, 23] investigated the ratio of the irredundance number and the domination number for different classes of graphs.

In this paper we disprove the known conjecture of Henning [3, 11] that a graph G is irredundance perfect if and only if it is 4-irredundance perfect. Moreover, the irredundant set problem and the dominating set problem are shown to be NP-complete on some classes of graphs. In particular, these problems are both NP-complete for irredundance perfect graphs. A number of problems and conjectures are proposed.

2 Summary of Known Results on Irredundance Perfect graphs

The following known properties of maximal irredundant sets are not redundant here:

Proposition 1 (Bollobás and Cockayne [2]) *Suppose that a vertex $u \in V(G)$ is not dominated by the maximal irredundant set X of G. Then for some $x \in X$,*

(a) $PN(x,X) \subseteq N(u)$

and

(b) for distinct vertices $x_1, x_2 \in PN(x,X)$, either $x_1x_2 \in E(G)$ or for $i = 1,2$ there exists $y_i \in X - \{x\}$ such that x_i is adjacent to each vertex of $PN(y_i, X)$.

While Proposition 1 gives necessary conditions for an irredundant set to be maximal, the next statement provides both necessary and sufficient condition for such a property.

Proposition 2 (Volkmann and Zverovich [18]) *Let X be an irredundant set of G, and $U = V(G) - N[X]$. The set X is a maximal irredundant set if and only if for any $v \in N[U]$, the vertex v dominates $PN(x,X)$ for some vertex $x \in X$.*

Proof: Suppose to the contrary that v does not dominate $PN(x,X)$ for any $x \in X$ and consider the set $X' = X \cup \{v\}$. Since $PN(x,X) \neq \emptyset$ and v does not dominate $PN(x,X)$, we have $PN(x,X') \neq \emptyset$ for any vertex $x \in X$. If $v \in U$, then $v \in PN(v,X')$. If $v \notin U$, then $v \perp u$ for some vertex $u \in U$ and hence $u \in PN(v,X')$. In any case, $PN(v,X') \neq \emptyset$. Thus, the set X' is irredundant in H. This is a contradiction, since X is maximal irredundant.

To prove the sufficiency, let u be an arbitrary vertex of $V(G) - X$. If $u \in N[U]$, then u dominates $PN(x,X)$ for some $x \in X$. Therefore, $PN(x, X \cup \{u\}) = \emptyset$. Suppose now that $u \notin N[U]$. We obtain $N[u] \subseteq N[X]$. Consequently, $PN(u, X \cup \{u\}) = \emptyset$. Thus, for any vertex $u \in V(G) - X$, the set $X \cup \{u\}$ is not irredundant. We conclude that X is a maximal irredundant set. ∎

The first result on irredundance perfect graphs is due to Bollobás and Cockayne.

Theorem 1 (Bollobás and Cockayne [2]) *If a graph G does not have two induced subgraphs isomorphic to P_4 with vertex sequences (a_i, b_i, c_i, d_i), $i = 1,2$, where $b_1, b_2, c_1, c_2, d_1, d_2$ are distinct and $a_i \notin \{c_1, c_2, d_1, d_2\}$ for $i = 1,2$, then G is an irredundance perfect graph.*

The following result of Favaron improves Theorem 1, since the graphs forbidden in Theorem 2 belong to the family of forbidden graphs of Theorem 1.

Theorem 2 (Favaron [7]) *If a graph G does not contain the graphs P_6, C_6, $2P_4$ and G_1, G_2, G_3 in Fig.1 as induced subgraphs, then G is irredundance perfect.*

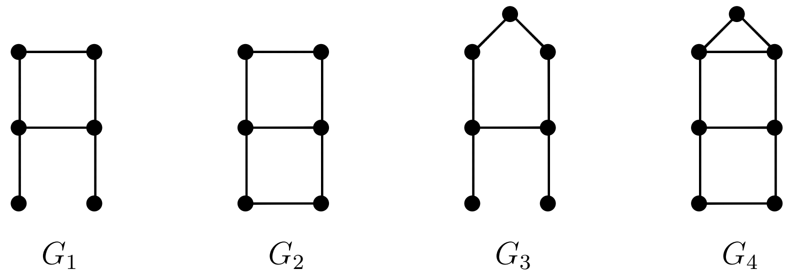

Fig.1. Graphs $G_1 - G_4$.

Favaron conjectured that only three graphs from six forbidden graphs described in Theorem 2 are needed as forbidden subgraphs for an irredundance perfect graph.

Conjecture 1 (Favaron [3, 7]) *If a graph G does not contain the graphs P_6 and G_1, G_2 in Fig.1 as induced subgraphs, then G is irredundance perfect.*

Henning proved that if G satisfies the conditions of Conjecture 1, then G belongs to $\mathcal{IP}_4$, a superclass of irredundance perfect graphs.

Theorem 3 (Henning [11]) *If a graph G does not contain the graphs P_6 and G_1, G_2 in Fig.1 as induced subgraphs, then G is a 4-irredundance perfect graph.*

Conjecture 1 follows from the next theorem, since the graph G_2 in Fig.1 is an induced subgraph of the graph G_4 in Fig.1. Moreover, Theorem 4 implies Theorems 1, 2 and 3.

Theorem 4 (Volkmann and Zverovich [18, 19]) *If a graph G does not contain the graphs P_6 and G_1, G_4 in Fig.1 as induced subgraphs, then G is an irredundance perfect graph.*

Conjecture 1 was independently proved by Puech [16] who also proved the following theorem.

Theorem 5 (Puech [16]) *If a graph G does not contain the induced graphs P_6 and G'_4,where G'_4 is obtained from G_4 by deleting the right lower vertex, then G is an irredundance perfect graph.*

Theorem 5 immediately implies the conjecture due to Faudree, Favaron and Li.

Conjecture 2 (Faudree, Favaron and Li [6]) *Every P_5-free graph is irredundance perfect.*

Puech [16] proposed the next conjecture. This conjecture, if true, would imply both Conjecture 1 and Conjecture 2. Let us denote by G'_3 the graph obtained from G_3 by adding the edge joining the two nonajacent vertices of degree 2.

Conjecture 3 (Puech [16]) *Every (P_6, G'_3, G_4)-free graph is irredundance perfect.*

Conjecture 3 was recently proved in [20].

Laskar and Pfaff also obtained a number of interesting results on irredundance perfect graphs.

Theorem 6 (Laskar and Pfaff [14]) *A chordal graph G is irredundance perfect if and only if G does not contain the Slater tree in Fig. 2 and H_1 in Fig.3 as induced subgraphs.*

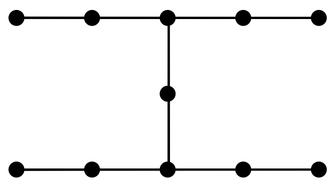

Fig.2. Slater tree.

Theorem 7 (Laskar and Pfaff [15]) *If G is the complement of a bipartite graph or a split graph and is connected, then $ir(G) = \gamma(G) = \gamma_t(G) = \gamma_c(G)$, where γ_t and γ_c are respectively the total and connected domination numbers of G.*

They also proved a sufficient condition for a graph to be both irredundance and domination perfect. Let $\mathcal{A}$ denote the family of graphs obtained from the graph H_1 in Fig.3 by adding any combinations of edges from the set $\{xy, xv, xz, yu, yz\}$.

Theorem 8 (Laskar and Pfaff [14]) *If G contains no induced $K_{1,3}$ and no induced graph from the family $\mathcal{A}$, then G is both irredundance and domination perfect.*

Theorem 8 was essentially improved by Favaron.

Theorem 9 (Favaron [7]) *If G does not contain $K_{1,3}$ and H_1 in Fig.3 as induced subgraphs, then G is both irredundance and domination perfect.*

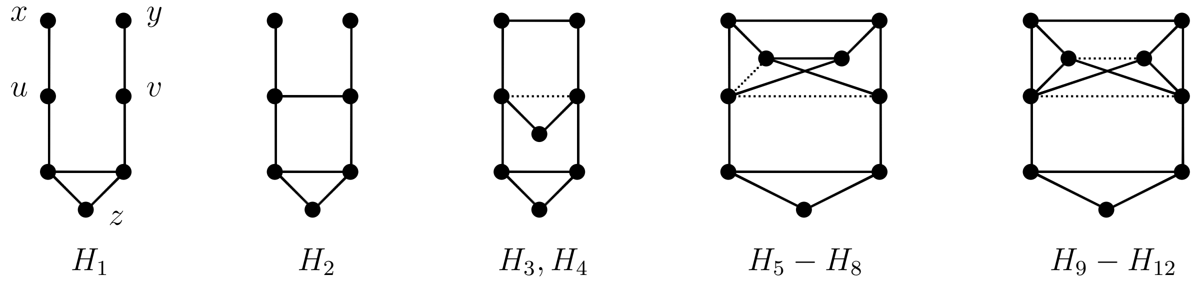

Fig.3. Henning graphs $H_1 - H_{12}$.

Henning found all minimal irredundance imperfect graphs having irredundance number two, those graphs are shown in Fig.3.

Theorem 10 (Henning [11]) *A graph G is 2-irredundance perfect if and only if G does not contain the graphs $H_1 - H_{12}$ in Fig.3 as induced subgraphs.*

Note that the original version of Theorem 10 was stated with superfluous graphs. Henning also posed an interesting conjecture that the class of irredundance perfect graphs coincides with the class $\mathcal{IP}_4$. This conjecture, if true, would give another proof of Conjecture 1 by Theorem 3.

Conjecture 4 (Henning [3, 11]) *A graph G is irredundance perfect if and only if G is 4-irredundance perfect, i.e., $ir(H) = \gamma(H)$ for every induced subgraph H of G with $ir(H) \leq 4$.*

We will see in the next section that Henning's conjecture is not true, i.e., the class of irredundance perfect graphs is a strict subclass of 4-irredundance perfect graphs.

3 Counterexample to Henning's Conjecture

In this section we prove that the graph F^* of Fig.4 is a counterexample to Conjecture 4, since F^* is 4-irredundance perfect but it is not irredundance perfect. This counterexample was first announced in [19]. Using a computer search, we discovered that F^* is a minimal irredundance imperfect graph.

Theorem 11 *The graph F^* of Fig.4 is a 4-irredundance perfect graph, and $ir(F^*) = 5$, $\gamma(F^*) = 6$.*

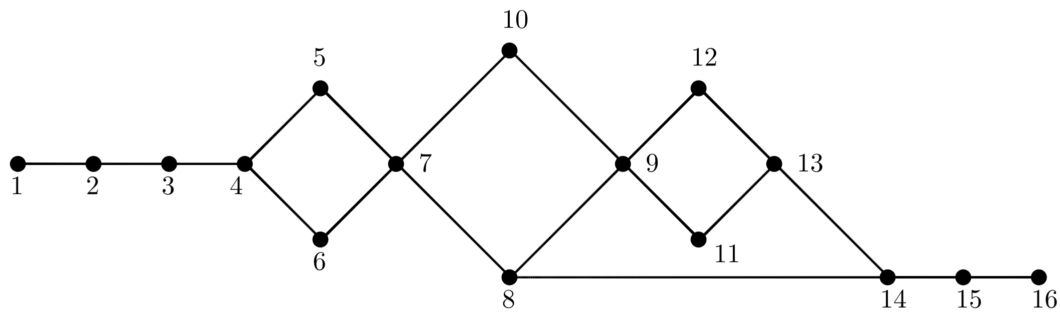

Fig.4. Counterexample F^* to Conjecture 4.

Proof: We need the following lemma.

Lemma 1 *If X is an ir-set of a minimal irredundance imperfect graph H, then the graph $\langle X \rangle$ has no isolated vertex, i.e., $x \notin PN(x, X)$ for any vertex $x \in X$.*

Proof: Let v be an isolated vertex in $\langle X \rangle$. Denote $H' = H - N[v]$ and $X' = X - \{v\}$. Obviously, for any vertex $x \in X'$,

$$PN_{H'}(x, X') = PN_H(x, X) \neq \emptyset.$$

Therefore, X' is an irredundant set in H'. Suppose that there is a vertex $u \in V(H') - X'$ such that the set $X' \cup \{u\}$ is irredundant in H', i.e., $PN_{H'}(x, X' \cup \{u\}) \neq \emptyset$ for any vertex $x \in X' \cup \{u\}$. It is not difficult to see that

$$PN_H(x, X \cup \{u\}) = PN_{H'}(x, X' \cup \{u\}) \neq \emptyset$$

for any $x \in X' \cup \{u\}$. Moreover, $PN_H(v, X \cup \{u\}) \neq \emptyset$, since $v \in PN_H(v, X \cup \{u\})$. We conclude that $X \cup \{u\}$ is an irredundant set in H, contrary to the fact that X is maximal irredundant. Consequently, X' is a maximal irredundant set in H' and hence

$$ir(H') \leq |X'| = ir(H) - 1.$$

If D is a minimum dominating set of H', then $D \cup \{v\}$ is a dominating set of H. Therefore,

$$\gamma(H) \leq |D| + 1 = \gamma(H') + 1.$$

We obtain

$$ir(H') \leq ir(H) - 1 < \gamma(H) - 1 \leq \gamma(H').$$

Thus, $ir(H') < \gamma(H')$, contrary to the minimality of the graph H. ∎

We first prove that F^* is a 4-irredundance perfect graph. Suppose to the contrary that this is not the case and consider the smallest induced subgraph H of F^* such that $ir(H) \leq 4$ and $ir(H) < \gamma(H)$. The graph H is, obviously, a minimal irredundance imperfect graph. Let X denote an ir-set of H. Since H has no triangle, we conclude by Theorem 10 that $ir(H) \geq 3$. Thus, there are two cases to consider.

Case 1 $ir(H) = 3$. We have $\gamma(H) > 3$. By Lemma 1, $\langle X \rangle$ is isomorphic to P_3. Let us denote by x the central vertex of the $\langle X \rangle$, and $U = V(H) - N_H[X]$. By Proposition 2, every vertex of U is adjacent to a vertex of $N_H(X) - X$. The vertex x has a nonempty private neighborhood in H. Therefore, $\deg_H(x) \geq 3$ and x cannot be one of the vertices 1, 2, 3, 5, 6, 10, 11, 12, 15, or 16. We should consider 6 subcases.

Case 1.1 $x = 4$. Assume that $X = \{3, 4, 5\}$. Obviously $2, 6, 7 \in V(H)$ and the set $\{2, 6, 7\}$ dominates H, a contradiction. The case $X = \{3, 4, 6\}$ is analogous. If $X = \{5, 4, 6\}$, then $PN_H(5, X) = \emptyset$, a contradiction.

Case 1.2 $x = 7$. Since X cannot contain redundant vertices, the cases $X = \{5, 7, 6\}$ and $X = \{8, 7, 10\}$ are impossible. Assume that $X = \{5, 7, 10\}$. If $14 \in V(H)$, then $6 \notin V(H)$ by Proposition 2, and $\{4, 8, 9\}$ dominates H, a contradiction. If $14 \notin V(H)$, then $\{4, 7, 9\}$ dominates H, a contradiction. Suppose that $X = \{5, 7, 8\}$. If $9, 14 \in V(H)$, then $11, 12, 13, 15 \notin V(H)$ by Proposition 2. Now $\{4, 7, 8\}$ dominates H, a contradiction. The vertex 8 is not redundant in X, and hence either 9 or 14 is present in H. This vertex together with $\{4, 7\}$ dominates H, a contradiction. The cases $X = \{6, 7, 10\}$ and $X = \{6, 7, 8\}$ are analogous.

Case 1.3 $x = 8$. Assume that $X = \{7, 8, 9\}$. If $4 \notin V(H)$, then $\{7, 14, 9\}$ dominates H. Hence $4 \in V(H)$. If 5 and 6 are present in H, then, by Proposition 2, the vertex 5 must dominate some private neighborhood, a contradiction. Therefore, either 5 or 6 is present and this vertex together with $\{14, 9\}$ dominates H, a contradiction. Consider the case $X = \{7, 8, 14\}$. By Proposition 2, if $16 \in V(H)$, then $13 \notin V(H)$. Also, if $4 \in V(H)$, then $10 \notin V(H)$ and exactly one vertex from the set $\{5, 6\}$ is present in H, say 5 is present. Now, we put $x = 5$ if $4 \in V(H)$, and we put $x = 7$ if $4 \notin V(H)$. Put $y = 15$ if $16 \in V(H)$ and $y = 14$ otherwise. It is easy to see that $\{x, y, 9\}$ dominates H, a contradiction. Suppose that $X = \{9, 8, 14\}$. If $16 \notin V(H)$, then $\{9, 7, 14\}$ dominates H. Otherwise, by Proposition 2, we have $13 \notin V(H)$ and $\{9, 7, 15\}$ dominates H.

Case 1.4 $x = 9$. If $X = \{10, 9, 11\}$, then $\{7, 9, 13\}$ dominates H. Suppose that $X = \{8, 9, 11\}$. If $7, 14 \in V(H)$, then $\{7, 14, 9\}$ dominates H. If exactly one vertex from $\{7, 14\}$ is present, then this vertex and $\{9, 11\}$ dominates H. The cases $X = \{10, 9, 12\}$ and $X = \{8, 9, 12\}$ are analogous. If $X = \{10, 9, 8\}$, then 10 is redundant in X. At last, if $X = \{11, 9, 12\}$, then 11 is redundant in X, a contradiction.

Case 1.5 $x = 13$. If $X = \{11, 13, 12\}$, then $PN_H(11, X) = \emptyset$, a contradiction. Assume that $X = \{11, 13, 14\}$. If $16 \in V(H)$, then $8, 7 \notin V(H)$ by Proposition 2, and $\{9, 12, 15\}$ dominates H. If $7 \in V(H)$, then $15, 16 \notin V(H)$ by Proposition 2. Now $\{9, 12, 8\}$ dominates H. If $16, 7 \notin V(H)$, then $\{9, 12, 14\}$ dominates H. The case $X = \{12, 13, 14\}$ is analogous.

Case 1.6 $x = 14$. If $X = \{13, 14, 15\}$, then $\{13, 8, 16\}$ dominates H. Assume that $X = \{8, 14, 15\}$. If $9 \notin V(H)$, then $7 \in V(H)$ and $\{7, 13, 16\}$ dominates H. If $7 \notin V(H)$, then $9 \in V(H)$ and $\{9, 13, 16\}$ dominates H. Suppose that $9, 7 \in V(H)$. Neither 9 nor 7 dominates some private neighborhood. By Proposition 2, we have $5, 6, 10 \notin V(H)$ and hence $\{8, 13, 16\}$ dominates H. Consider the case $X = \{8, 14, 13\}$. If $5, 6, 10 \notin V(H)$, then $\{8, 15, 13\}$ dominates H. Hence some of those vertex is present in H. If $7, 9 \in V(H)$, then 7 must dominate a private neighborhood by Proposition 2, a contradiction. Therefore, exactly one vertex from $\{7, 9\}$ is present in H. This vertex together with $\{15, 13\}$ dominates H, a contradiction.

Case 2 $ir(H) = 4$. We have $\gamma(H) > 4$. Since H is a minimal irredundance imperfect graph, it follows that H is connected. Put $U = V(H) - N_H[X]$. By Proposition 2, every vertex of U is adjacent to a vertex of $N_H(X) - X$. The graph H is triangle-free. Hence, by Lemma 1, $\langle X \rangle$ is isomorphic to $C_4, K_{1,3}, P_4$ or $2K_2$. If $\langle X \rangle$ is isomorphic to C_4, then X contains a redundant vertex, a contradiction. Suppose that $\langle X \rangle$ is isomorphic to $K_{1,3}$ and denote by x its central vertex. Since x is not redundant in X, it follows that $\deg_H(x) \geq 4$. Hence either $x = 7$ or $x = 9$. It is not difficult to see that, in any of these cases, the set X contains a redundant vertex, a contradiction. It remains to consider two cases.

Case 2.1 $\langle X \rangle \cong P_4$. Denote by e the central edge of the P_4. Since X has no redundant vertex, the endvertices of e have degree at least 3. It follows that e is one of the edges $(7, 8), (8, 9), (8, 14)$ or $(13, 14)$.

Subcase 2.1.1 $e = (7, 8)$. If $X = \{6, 7, 8, 9\}$, then $\{4, 7, 14, 9\}$ dominates H. Suppose that $X = \{6, 7, 8, 14\}$. If $16 \in V(H)$, then $13 \notin V(H)$ by Proposition 2, and $\{4, 7, 9, 15\}$ dominates H. If $16 \notin V(H)$, then $\{4, 7, 9, 14\}$ dominates H. The cases $X = \{5, 7, 8, 9\}$ and $X = \{5, 7, 8, 14\}$ are analogous. If $X = \{10, 7, 8, 14\}$, then $PN_H(10, X) = \emptyset$.

Subcase 2.1.2 $e = (8, 9)$. Suppose that $X = \{7, 8, 9, 11\}$. If $4 \notin V(H)$, then $\{7, 14, 9, 11\}$ dominates H. Hence $4 \in V(H)$. If $5, 6 \in V(H)$, then 5 must dominate some private neighborhood by Proposition 2, a contradiction. Therefore, exactly one vertex from $\{5, 6\}$ is present in H, and this vertex together with $\{14, 9, 11\}$ dominates H. The case $X = \{7, 8, 9, 12\}$ is analogous. The cases $X = \{14, 8, 9, 10\}$, $X = \{14, 8, 9, 11\}$ and $X = \{14, 8, 9, 12\}$ are impossible because of a redundant vertex in X.

Subcase 2.1.3 $e = (8, 14)$. If $X = \{9, 8, 14, 13\}$, then 13 is redundant in X. If $X = \{9, 8, 14, 15\}$, then $\{9, 7, 14, 15\}$ dominates H. Suppose that $X = \{7, 8, 14, 13\}$. If $4 \notin V(H)$, then $\{7, 9, 15, 13\}$ dominates H. Hence $4 \in V(H)$. If $5, 6 \in V(H)$, then 5 must dominate some private neighborhood by Proposition 2, a contradiction. Therefore, exactly one vertex from $\{5, 6\}$ is present in H, and this vertex together with $\{9, 15, 13\}$ dominates H. The case $X = \{7, 8, 14, 15\}$ is considered analogously.

Subcase 2.1.4 $e = (13, 14)$. The cases $X = \{11, 13, 14, 8\}$ and $X = \{12, 13, 14, 8\}$ are impossible because of a redundant vertex in X. If $X = \{11, 13, 14, 15\}$ or $X = \{12, 13, 14, 15\}$, then $\{9, 13, 8, 16\}$ dominates H, a contradiction.

Case 2.2 $\langle X \rangle \cong 2K_2$. Denote by e and f the edges of the $2K_2$. Since X has no redundant vertex, neither e or f can be $(1, 2)$ or $(15, 16)$. Taking into account the symmetry of the edges $(4, 5)$ and $(4, 6)$, $(5, 7)$ and $(6, 7)$, $(9, 11)$ and $(9, 12)$, $(11, 13)$ and $(12, 13)$, there are 37 cases to consider.

Subcase 2.2.1 $e = (2, 3), f = (5, 7)$. We have $PN_H(3, X) = \emptyset$, a contradiction.

Subcase 2.2.2 $e = (2, 3), f = (7, 10)$. If $14 \notin V(H)$, then $\{2, 3, 7, 9\}$ dominates H. If $14 \in V(H)$, then $5, 6 \notin V(H)$ by Proposition 2, and $\{2, 3, 8, 9\}$ dominates H, a contradiction.

Subcase 2.2.3 $e = (2, 3), f = (7, 8)$. If 11 or 12 is present in H, then $14 \notin V(H)$ by Proposition 2, and $\{2, 3, 7, 9\}$ dominates H. If 13 or 15 is present in H, then $9 \notin V(H)$ by Proposition 2, and $\{2, 3, 7, 14\}$ dominates H. If none of the vertices $11, 12, 13, 15$ is present, then $\{2, 3, 7, 8\}$ dominates H, a contradiction.

Subcase 2.2.4 $e = (2, 3), f = (8, 9)$. Since H is connected, $7 \in V(H)$. By Proposition 2, we have $15 \notin V(H)$. If $13 \notin V(H)$, then $\{2, 4, 8, 9\}$ dominates H, and so $13 \in V(H)$. By Proposition 2, the vertex 13 must dominate $PN_H(9, X)$. Therefore, $10 \notin V(H)$. If $14 \in V(H)$, then it must dominate a private neighborhood by Proposition 2, a contradiction. Thus, $14 \notin V(H)$ and $\{2, 4, 8, 13\}$ dominates H, a contradiction.

Subcase 2.2.5 $e = (2, 3), f = (9, 10)$. If $13 \notin V(H)$ and $14 \notin V(H)$, then $\{2, 4, 10, 9\}$ dominates H. If $14 \in V(H)$, then $11, 12, 13 \notin V(H)$ by Proposition 2, and $\{2, 4, 7, 8\}$ dominates H. If $13 \in V(H)$, then $8, 14 \notin V(H)$ by Proposition 2, and $\{2, 4, 10, 13\}$ dominates H, a contradiction.

Subcase 2.2.6 $e = (2, 3), f = (8, 14)$. Since H is connected, $7 \in V(H)$. Suppose that $16 \in V(H)$, and hence $15 \in V(H)$. By Proposition 2, we have $13 \notin V(H)$ and $11, 12 \notin V(H)$. If $10 \in V(H)$, then, by Proposition 2, we obtain $9 \notin V(H)$ and $\{2, 4, 7, 15\}$ dominates H. If $10 \notin V(H)$, then $\{2, 4, 8, 15\}$ dominates H, a contradiction.

Now assume that $16 \notin V(H)$. Consider the case $10 \notin V(H)$. If $11, 12 \notin V(H)$, then $\{2, 4, 8, 14\}$ dominates H, and hence one of those vertices, say 11, is present in H. Since 11 must dominate a private neighborhood, $15 \notin V(H)$ and $13 \in V(H)$, which produces the dominating set $\{2, 4, 8, 13\}$. Suppose now that $10 \in V(H)$. Since the vertex 7 must dominate a private neighborhood, we have $9 \notin V(H)$. If $15 \in V(H)$, then $11, 12 \notin V(H)$

by Proposition 2, and $\{2,4,7,14\}$ dominates H. If $15 \notin V(H)$, then $13 \in V(H)$ and $\{2,4,7,13\}$ dominates H, a contradiction.

Subcase 2.2.7 $e = (2,3), f = (9,11)$. Since H is connected, $7 \in V(H)$. By Proposition 2, the vertex 7 must dominate a private neighborhood, and so $12 \notin V(H)$. By the same argument, only one vertex from $\{10,8\}$ is present in H, say $10 \in V(H)$. Now $\{2,4,10,13\}$ dominates H, a contradiction.

Subcase 2.2.8 $e = (2,3), f = (11,13)$. The graph H is disconnected, a contradiction.

Subcase 2.2.9 $e = (2,3), f = (13,14)$. Since H is connected, $7,8 \in V(H)$. We have $15,16 \notin V(H)$ by Proposition 2, and $\{2,4,8,13\}$ dominates H, a contradiction.

Subcase 2.2.10 $e = (2,3), f = (14,15)$. Since H is connected, $7,8 \in V(H)$. By Proposition 2, $11,12,13 \notin V(H)$. Now $\{2,4,8,15\}$ dominates H, a contradiction.

Subcase 2.2.11 $e = (3,4), f = (7,10)$. We have $PN_H(4,X) = \emptyset$, a contradiction.

Subcase 2.2.12 $e = (3,4), f = (7,8)$. We have $PN_H(4,X) = \emptyset$, a contradiction.

Subcase 2.2.13 $e = (3,4), f = (8,9)$. Since H is connected, $7 \in V(H)$. By Proposition 2, we have $15 \notin V(H)$. Also, $13 \in V(H)$, for otherwise $\{2,4,8,9\}$ dominates H. By Proposition 2, we obtain $10 \notin V(H)$. Now $\{2,4,8,13\}$ dominates H, a contradiction.

Subcase 2.2.14 $e = (3,4), f = (9,10)$. If $13 \in V(H)$, then $8 \notin V(H)$ by Proposition 2. Now $\{3,4,10,13\}$ dominates H. Hence $13 \notin V(H)$. If $14 \notin V(H)$, then $\{3,4,10,9\}$ dominates H. If $14 \in V(H)$, then $11,12 \notin V(H)$ by Proposition 2. We have $\{3,4,10,8\}$ dominates H, a contradiction.

Subcase 2.2.15 $e = (3,4), f = (8,14)$. Since H is connected, we have $7 \in V(H)$. Suppose that $9 \in V(H)$. The vertex 9 does not dominate a private neighborhood, and hence, by Proposition 2, we obtain $10,11,12 \notin V(H)$. If $16 \notin V(H)$, then $\{2,4,8,14\}$ dominates H. If $16 \in V(H)$, then $13 \notin V(H)$ by Proposition 2, and $\{2,4,8,15\}$ dominates H, a contradiction.

Assume that $9 \notin V(H)$. If 11 or 12 is present in H, then $15,16 \notin V(H)$ by Proposition 2, and $\{2,4,7,13\}$ dominates H. If 16 is present in H, then $11,12,13 \notin V(H)$ by Proposition 2, and $\{2,4,7,15\}$ dominates H. If none of the vertices $11,12,16$ is present, then $\{2,4,7,14\}$ dominates H, a contradiction.

Subcase 2.2.16 $e = (3,4), f = (9,11)$. Since H is connected, we have $7 \in V(H)$. By Proposition 2, only one vertex from $\{5,6\}$ is present in H, and this vertex together with $\{2,9,13\}$ dominates H, a contradiction.

Subcase 2.2.17 $e = (3,4), f = (11,13)$. Since H is connected, we have $7 \in V(H)$. By Proposition 2, only one vertex from $\{5,6\}$ is present in H, say $5 \in V(H)$. Put $D = \{2,5,9,14\}$ if $14 \in V(H)$, and $D = \{2,5,9,13\}$ otherwise. The set D dominates H, a contradiction.

Subcase 2.2.18 $e = (3,4), f = (13,14)$. Since H is connected, we have $7,8 \in V(H)$. By Proposition 2, we have $16 \notin V(H)$. Also, $15 \notin V(H)$, since 8 must dominate a private neighborhood. Again, by Proposition 2, only one vertex from $\{5,6\}$ is present in H, say $5 \in V(H)$. The set $\{2,5,8,13\}$ dominates H, a contradiction.

Subcase 2.2.19 $e = (3,4), f = (14,15)$. Since H is connected, we have $7,8 \in V(H)$. By Proposition 2, we obtain $11,12 \notin V(H)$, and $13 \notin V(H)$, since 8 must dominate a private neighborhood. Again, by Proposition 2, only one vertex from $\{5,6\}$ is present in H, say $5 \in V(H)$. The set $\{2,5,8,15\}$ dominates H, a contradiction.

Subcase 2.2.20 $e = (4,5), f = (8,9)$. We have $PN_H(5,X) = \emptyset$, a contradiction.

Subcase 2.2.21 $e = (4, 5), f = (9, 10)$. We have $PN_H(5, X) = \emptyset$, a contradiction.

Subcase 2.2.22 $e = (4, 5), f = (8, 14)$. We have $PN_H(5, X) = \emptyset$, a contradiction.

Subcase 2.2.23 $e = (4, 5), f = (9, 11)$. The set $\{3, 7, 9, 13\}$ dominates H if $3 \in V(H)$, and $\{4, 7, 9, 13\}$ dominates H otherwise, a contradiction.

Subcase 2.2.24 $e = (4, 5), f = (11, 13)$. We put $x = 3$ if $3 \in V(H)$, and $x = 4$ otherwise. Also, put $y = 14$ if $14 \in V(H)$, and $y = 13$ otherwise. The set $\{x, y, 7, 9\}$ dominates H, a contradiction.

Subcase 2.2.25 $e = (4, 5), f = (13, 14)$. Put $x = 3$ if $3 \in V(H)$, and put $x = 4$ otherwise. Also, put $y = 15$ if $15 \in V(H)$, and put $y = 14$ otherwise. If $9 \notin V(H)$, then $\{x, y, 7, 13\}$ dominates H. Hence $9 \in V(H)$. By Proposition 2, only one vertex from $\{11, 12\}$ is present in H, say $11 \in V(H)$. The set $\{x, y, 7, 11\}$ dominates H, a contradiction.

Subcase 2.2.26 $e = (4, 5), f = (14, 15)$. Put $x = 3$ if $3 \in V(H)$, and put $x = 4$ otherwise. Also, put $y = 13$ if $13 \in V(H)$, and put $y = 14$ otherwise. If $9 \notin V(H)$, then $\{x, y, 7, 15\}$ dominates H. Hence $9 \in V(H)$. By Proposition 2, we have $13 \notin V(H)$ and therefore $11, 12 \notin V(H)$. The set $\{x, 7, 8, 15\}$ dominates H, a contradiction.

Subcase 2.2.27 $e = (5, 7), f = (9, 11)$. The set $\{4, 7, 9, 13\}$ dominates H, a contradiction.

Subcase 2.2.28 $e = (5, 7), f = (11, 13)$. If $15 \notin V(H)$, then $\{4, 7, 11, 13\}$ dominates H. If $15 \in V(H)$, then $\{4, 7, 9, 14\}$ dominates H, a contradiction.

Subcase 2.2.29 $e = (5, 7), f = (13, 14)$. If $9 \notin V(H)$, then $\{4, 7, 13, 15\}$ dominates H. Hence $9 \in V(H)$. By Proposition 2, only one vertex from $\{11, 12\}$ is present in H, say $11 \in V(H)$. Now the set $\{4, 7, 11, 15\}$ dominates H, a contradiction.

Subcase 2.2.30 $e = (5, 7), f = (14, 15)$. If $9 \notin V(H)$, then $\{4, 7, 13, 15\}$ dominates H. Hence $9 \in V(H)$. By Proposition 2, we have $6, 8 \notin V(H)$. The set $\{4, 10, 13, 15\}$ dominates H, a contradiction.

Subcase 2.2.31 $e = (7, 10), f = (11, 13)$. We have $PN_H(10, X) = \emptyset$, a contradiction.

Subcase 2.2.32 $e = (7, 10), f = (13, 14)$. If $4 \notin V(H)$, then $\{7, 10, 13, 15\}$ dominates H. If $4 \in V(H)$, then, by Proposition 2, only one vertex from $\{5, 6\}$ is present in H, say $5 \in V(H)$. Then $\{5, 9, 13, 15\}$ dominates H, a contradiction.

Subcase 2.2.33 $e = (7, 10), f = (14, 15)$. If $4 \notin V(H)$, then $\{7, 9, 13, 15\}$ dominates H. If $4 \in V(H)$, then, by Proposition 2, only one vertex from $\{5, 6\}$ is present in H, say $5 \in V(H)$. Then $\{5, 9, 13, 15\}$ dominates H, a contradiction.

Subcase 2.2.34 $e = (7, 8), f = (11, 13)$. We have $PN_H(11, X) = \emptyset$, a contradiction.

Subcase 2.2.35 $e = (9, 10), f = (13, 14)$. We have $PN_H(13, X) = \emptyset$, a contradiction.

Subcase 2.2.36 $e = (9, 10), f = (14, 15)$. The set $\{7, 9, 14, 15\}$ dominates H, a contradiction.

Subcase 2.2.37 $e = (9, 11), f = (14, 15)$. We have $PN_H(11, X) = \emptyset$, a contradiction.

Thus, F^* is a 4-irredundance perfect graph. Let us show that $\gamma(F^*) = 6$. Clearly, there is a minimum dominating set D of F^* such that $2, 15 \in D$. Suppose that $7 \in D$. To dominate 4 we need one vertex, and to dominate $C_4 = \langle 9, 11, 12, 13 \rangle$ we need two more vertices. Therefore, $|D| \geq 6$. Assume now that $7 \notin D$. If $4 \notin D$, then $5, 6 \in D$ and we need more two vertices to dominate the above C_4, i.e. $|D| \geq 6$. Consider the case $4 \in D$. There are two possibilities. If $10 \in D$, then we need more two vertices to dominate the set $\{8, 11, 12, 13\}$. If $10 \notin D$, then $9 \in D$, for otherwise 10 is not dominated.

Now we need two more vertices to dominate $\{7, 13\}$. Therefore, $\gamma(F^*) = |D| \geq 6$. Since $\{2, 4, 7, 9, 13, 15\}$ is a dominating set, we obtain $\gamma(F^*) = 6$.

Now we prove that $ir(F^*) = 5$. For the set $X = \{3, 4, 8, 13, 14\}$, we have

$$PN(3) = \{2\}, \quad PN(4) = \{5, 6\},$$

$$PN(8) = \{7, 9\}, \quad PN(13) = \{11, 12\}, \quad PN(14) = \{15\}.$$

Therefore, X is an irredundant set. To prove that X is a maximal irredundant set, we apply Proposition 2. We have, $U = \{1, 10, 16\}$, $N[U] = U \cup \{2, 7, 9, 15\}$, and 1 dominates $PN(3)$, 10 dominates $PN(8)$, 16 dominates $PN(14)$, 2 dominates $PN(3)$, 7 dominates $PN(4)$, 9 dominates $PN(13)$ and 15 dominates $PN(14)$. By Proposition 2, X is maximal irredundant, and so $ir(F^*) \leq |X| = 5$. If $ir(F^*) < 5$, then we have a contradiction, since F^* is a 4-irredundance perfect graph and $\gamma(F^*) = 6$. Thus, $ir(F^*) = 5$. ∎

4 Complexity Results

To prove the result on NP-completeness of the irredundant set and dominating set problems, we need the following improvement of Theorem 6.

Theorem 12 *If G does not contain the Slater tree in Fig.2, the graph H_1 in Fig.3, and the cycles C_4, C_5, C_6, C_7 as induced subgraphs, then G is irredundance perfect.*

Proof: It is sufficient to prove that $ir(G) = \gamma(G)$. Suppose that $ir(G) < \gamma(G)$ and consider an ir-set X. The set X is not dominating, and so $U = V(G) - N[X] \neq \emptyset$. Denote

$$X^+ = \{x \in X : PN(x, X) \subseteq N(u) \text{ for some } u \in U\} \quad \text{and} \quad X^- = X - X^+.$$

For all $x \in X^+$, take one vertex from $PN(x)$ and form the set Y. Each pair of vertices from $PN(x)$, where $x \in X^+$, is adjacent to x and to a vertex from U, and G contains no induced C_4. Hence $\langle PN(x) \rangle$ is a complete graph for any $x \in X^+$. By Proposition 2, each vertex of U is adjacent to some vertex of Y. Consequently, the set $R = Y \cup X^-$ dominates

$$D = U \cup PN \cup X \cup N(X^-),$$

where

$$PN = \cup_{x \in X} PN(x, X).$$

Since

$$|R| = |X| = ir(G) < \gamma(G),$$

the set R does not dominate $v \in V(G)$, and hence $v \notin D$. We have $v \perp x_i$, where $x_i \in X^+$, $i = 1, 2$. For $i = 1, 2$, there are vertices $y_i \in Y \cap PN(x_i, X)$ such that $x_i \perp y_i$ and there are $u_i \in U$ such that $u_i \perp y_i$. Clearly, $v \pm \{y_1, y_2\}$, $y_1 \pm x_2$ and $y_2 \pm x_1$. Since G does not contain induced C_i $(i = 4, 5, 6, 7)$, we have $u_1 \neq u_2$, $u_1 \pm u_2$, $u_1 \pm y_2$, $u_2 \pm y_1$, $v \pm \{u_1, u_2\}$ and $y_1 \pm y_2$. If $x_1 \perp x_2$, then $\langle \{u_i, y_i, x_i, v : i = 1, 2\} \rangle \cong H_1$ in Fig.3, a contradiction. Hence $x_1 \pm x_2$. In fact, for the above vertex v, we proved the following lemma.

Lemma 2 *If the vertex v is adjacent to $a \in X^+$ and $b \in X^+$, then $a \pm b$.*

Suppose now that there is $x \in X$ such that $x \perp x_1$ and $x \perp x_2$. Since $\langle\{x_1, x_2, x, v\}\rangle \ncong C_4$, we have $v \perp x$. Moreover, $v \notin N(X^-)$, and hence $x \in X^+$. By Lemma 2, $x \pm x_1$, a contradiction.

The vertices x_1, x_2 are not isolated in $\langle X\rangle$ because $x_i \notin PN(x_i, X)$ for $i = 1, 2$. Therefore, for $i = 1, 2$, there exist $z_i \in X$ such that $z_i \perp x_i$. If $z_i \perp v$, then $z_i \in X^+$ and, by Lemma 2, $z_i \pm x_i$, a contradiction. Hence $z_1 \pm v$ and $z_2 \pm v$. Also, $z_1 \neq z_2$, $z_1 \pm x_2$ and $z_2 \pm x_1$, for otherwise we have induced C_4. The vertices z_1, z_2 are not isolated in $\langle X\rangle$, so there are $w_i \in PN(z_i, X)$, $i = 1, 2$. Obviously, $w_i \neq z_i$. Since the cycles C_i $(i = 4, 5, 6, 7)$ are forbidden, we see that $z_1 \pm z_2$, $w_1 \pm w_2$ and $w_i \pm \{u_j, y_j\}$, $i, j = 1, 2$. We obtain $\langle\{u_i, y_i, x_i, v, z_i, w_i : i = 1, 2\}\rangle$ is isomorphic to the Slater tree in Fig.2. This contradiction completes the proof of Theorem 12. ∎

We say that a graph G *belongs to the class* $\mathcal{L}$ if G is a bipartite planar graph of maximum degree 3 and girth $g(G) > |G|^k$, where k is fixed, $0 \leq k < 1$. In our next theorem, the irredundant set and dominating set problems are shown to be NP-complete on the class $\mathcal{L} \cap \mathcal{IP}$. Note that the irredundant set problem is known (see [10]) to be NP-complete for bipartite graphs and for chordal graphs only. The results on NP-completeness of the dominating set problem can be found in [5, 13, 21].

Theorem 14 *The irredundant set and dominating set problems are both NP-complete on the class of irredundance perfect graphs from $\mathcal{L}$.*

Proof: It is known (see [13]) that the dominating set problem is NP-complete for 3-regular planar graphs. We describe a polynomial time reduction from this problem to the irredundant set and dominating set problems for the class $\mathcal{L} \cap \mathcal{IP}$, which implies the desired result.

The operation of 3-*partition* of an edge is defined in the following way: replace an edge uv by the chain $P_5 = (u, x, y, z, v)$ with endvertices u and v. Suppose H is obtained from G by single 3-partition of an edge uv. Let us prove that

$$\gamma(H) = \gamma(G) + 1.$$

Let D be a minimum dominating set of G. If $u, v \in D$ or $u, v \notin D$, then $D \cup \{y\}$ dominates H. If $u \in D$ and $v \notin D$, then $D \cup \{z\}$ dominates H. The case $u \notin D$ and $v \in D$ is similar. Therefore $\gamma(H) \leq \gamma(G) + 1$. Let B be a minimum dominating set of H. Clearly, $S = B \cap \{u, x, y, z\} \neq \emptyset$. If $|S| = 1$, then $S = \{x\}$ or $S = \{y\}$, and in the case $S = \{x\}$ we have $v \in B$. Obviously, $B - S$ dominates G. If $|S| \geq 2$, then $(B - S) \cup \{u\}$ dominates G. Thus $\gamma(G) \leq \gamma(H) - 1$.

Define the operation of $3s$-*partition* of an edge uv as follows: uv is replaced by the chain $P_{3s+2} = (u, x_1, x_2, ..., x_{3s}, v)$. Let H be obtained from G by $3s$-partition of an edge. Since the $3s$-partition is a repetition of 3-partitions, we have

$$\gamma(H) = \gamma(G) + s.$$

Let G be a 3-regular planar graph. Choose a positive integer m such that $m/(m+2) \geq k$. Further, put $s = |G|^m$ if $|G|^m$ is odd, and put $s = |G|^m - 1$ if $|G|^m$ is even. Apply the operation of $3s$-partition to each edge of the graph G. Obviously, the resulting graph F is a planar graph of maximum degree 3. Since s is odd, F is bipartite. Taking into account that $|E(G)| = 1.5|G|$, we have for $|G| \geq 6$:

$$|F| = |G| + 3s|E(G)| = |G| + 4.5s|G| \leq |G| + 4.5|G|^{m+1} < |G|^{m+2}.$$

Hence

$$g(F) \geq 3 + 9s > |G|^m > |F|^{m/(m+2)} \geq |F|^k.$$

Thus, F is a member of $\mathcal{L}$. Moreover, the graph F does not contain the cycles C_i ($i = 4, 5, 6, 7$), the Slater tree in Fig.2, and the graph H_1 in Fig.3 as induced subgraphs. By Theorem 12, F is irredundance perfect and hence

$$ir(F) = \gamma(F) = \gamma(G) + s|E(G)|.$$

Since k is fixed, the reduction is computable in polynomial time. The proof of Theorem 14 is complete. ∎

The next three corollaries follow directly from Theorem 14.

Corollary 1 *The irredundant set and dominating set problems are both NP-complete on the class of irredundance perfect graphs.*

Corollary 2 *The irredundant set problem is NP-complete on the class $\mathcal{L}$.*

Corollary 3 (Emden-Weinert, Hougardy and Kreuter [5]) *The dominating set problem is NP-complete on the class $\mathcal{L}$.*

5 Problems and Conjectures

In spite of the fact that there are minimal irredundance imperfect graphs of orders 15, 16 and 17, we strongly believe that irredundance perfect graphs can be characterized in terms of a finite number of forbidden induced subgraphs. A proof or disproof of this conjecture would be a great contribution towards a characterization of irredundance perfect graphs.

Conjecture 5 *The number of minimal irredundance imperfect graphs is finite.*

Even though Henning's conjecture is not true, we did not find a counterexample to the following conjecture.

Conjecture 6 *A graph G is irredundance perfect if and only if G is 5-irredundance perfect.*

The problems worth investigating are presented below.

Problem 1 *Characterize the following classes of graphs:*

(a) 3-irredundance perfect graphs;
(b) 4-irredundance perfect graphs;
(c) K_3-free irredundance perfect graphs;
(d) C_4-free irredundance perfect graphs;
(e) P_6-free irredundance perfect graphs.

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