Nurse Rescheduling


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Background

- Scheduling nursing staff is complex
- Need “good” schedules for nurses
- Should take account of nurses' preferences when scheduling
- ... and disruption when rescheduling
- But ...
- little research on nurse rescheduling
Literature

- Nurse Scheduling
  - Preference-scheduling: Bard & Purnomo (2005ab)
  - Cyclic→Assign→Non-cyclic: Brucker et al (2005)
Literature

- Nurse Rescheduling
  - Little research on how shift changes affect nurses.
Aim of Presentation

- Develop 2 nurse scheduling models
  - Individual days
  - Shift patterns
- Adapt models for rescheduling, so that
  - Quality is maximised
  - Disruption is minimised
- Consider nurses preferences and fairness of shifts changes when (re)scheduling
Typical Problem

- 20 permanent nursing staff
- Planning horizon of 28 days (4 weeks)
- 3 shifts/day: *Early*, *Late*, *Night*
- All $28 \times 3 = 84$ shifts to be covered
- No floating nurses or on-call staff
- 3 contracts: *FullTime*, *PartTime*, *DayOnly*
- All nurses at same level and experience
Modelling & Solution Method

- Mixed Integer Programme (in AMPL)
- Model has multiple objectives
  - Main: minimise any nurse *shortages* on shifts
  - then: minimise *surplus* nurses on shifts
- Solvers:
  - Use Cplex v11.0, or Minto on Neos Server
  - Neos is free (via internet) and easy to use
  - Neos/Minto time limit of 10 min (self-imposed)
Messy Constraints

- Only 1 shift per day can be worked
- Cannot work early or late, after a night
- $\geq 2$ days off per week
- $\leq 2$ night shifts per week
- No more than FT/PT/DO contracted hours
- $\geq 1$ weekend off in 4
- Avoid dispersed individual days off
- Work $\leq 5$ days in a row
Data:

\[ r_{sdw} \] Number of nurses required on shift \( s \) on day \( d \) in week \( w \)

\[ h_s \] Number of hours in shift \( s \)

\[ H_c \] Maximum contracted hours for contract \( c \)

\( t(n) \) Type of contract of nurse \( n \)

Decision Variables:

\[ x_{n, sdw} \] Assigns nurse \( n \) to shift \( s \) on day \( d \) in week \( w \) (= 1, otherwise = 0)

\[ S^+_{sdw} \] Shortage of nurses on shift \( s \) of day \( d \) in week \( w \), \( \geq 0 \)

\[ S^-_{sdw} \] Surplus of nurses on shift \( s \) of day \( d \) in week \( w \), \( \geq 0 \)
Individual Days Model 2

Minimise \[ \sum_{s} \sum_{d} \sum_{w} \left( S_{sdw}^+ - 0.001S_{sdw}^- \right) \]

such that

\[ \sum_{n} x_{nsdw} + S_{sdw}^+ + S_{sdw}^- = r_{sdw} \quad \forall s, d, w \]

\[ \sum_{n} x_{nsdw} = 1 \quad \forall n, d, w \]

\[ \sum_{s} \sum_{d} x_{nsdw} \leq 5 \quad \forall n, w \]
\[ \sum_{d} x_{n, \text{Night}, d, w} \leq 2 \quad \forall n \mid t(n) \neq \text{DayOnly}, w \]

\[ x_{n, \text{Night}, d, w} = 0 \quad \forall n \mid t(n) = \text{DayOnly}, d, w \]

\[ x_{n, \text{Night}, d, w} + x_{n, \text{Early}, d+1, w} + x_{n, \text{Late}, d+1, w} = 1 \quad \forall n, d < 7, w \]

\[ x_{n, \text{Night}, 7, w} + x_{n, \text{Early}, 1, w+1} + x_{n, \text{Late}, 1, w+1} = 1 \quad \forall n, w < 4 \]

\[ x_{n, \text{Late}, d, w} + x_{n, \text{Early}, d+1, w} \leq 1 \quad \forall n, d < 7, w \]

\[ x_{n, \text{Late}, 7, w} + x_{n, \text{Early}, 1, w+1} \leq 1 \quad \forall n, w < 4 \]

\[ \sum_{s} \sum_{d} h_{s} x_{nsdw} \leq H_{c(n)} \quad \forall n, w \]
Individual-Days Model - Speed

- Normal capacity: 2 or 3 nurses per shift
  - Cplex: 4 secs to optimal zero solution
  - Neos: 10 minutes. Gap (from Cplex) = 100%
  - But uneven use of nurses.

- Tight capacity: 1 extra nurse per shift
  - Cplex: 10 minutes. Gap = 3%
  - Neos: 10 minutes. No feasible solution found.
Shift-Patterns Model 1

Data:

\[ r_{sdw} \] Number of nurses required on shift \( s \) on day \( d \) in week \( w \)

\[ c_{sdp} = 1 \text{ if pattern } p \text{ covers shift } s \text{ on day } d, \text{ otherwise } 0 \]

Decision Variables:

\[ x_{npw} \] Assigns nurse \( n \) to shift pattern \( p \) in week \( w \) (\( = 1 \), otherwise \( = 0 \)).

\[ S^+_{sdw} \] Shortage of nurses on shift \( s \) of day \( d \) in week \( w \), \( \geq 0 \).

\[ S^-_{sdw} \] Surplus of nurses on shift \( s \) of day \( d \) in week \( w \), \( \geq 0 \).
Shift-Patterns Model 2

Minimise \[ \sum_{s} \sum_{d} \sum_{w} \left( S_{sdw}^{+} + 0.001S_{sdw}^{-} \right) \]

such that

\[ \sum_{n} \sum_{p} c_{spd} x_{npw} + S_{sdw}^{+} - S_{sdw}^{-} = r_{sdw} \quad \forall s, d, w \]

\[ \sum_{p} x_{npw} = 1 \quad \forall n, w \]
Patterns model - speed

- Model chooses from just 26 possible 1-week shift patterns, so not exhaustively optimal.

- Normal capacity (2 or 3 nurses per shift)
  - Cplex: 0.23 secs to “optimal” (zero) solution
  - Neos: 1.62 secs to “optimal” (zero) solution

- Tight capacity (1 more nurse per shift)
  - Cplex: 33 secs to “optimal” (non-zero) solution.
  - Neos: 10 minute limit reached: Gap = 16.7%
Scaling up (tight capacity)

- 6 weeks, 20 nurses.  Cplex: 229 secs. Optimal  
  Neos: 10 min. Gap (from Cplex) = 6.3%

- 4 weeks, 30 nurses.  Cplex: 174 secs. Optimal  
  Neos: 10 min. Gap (from Cplex) = 7.6%

- 8 weeks, 20 nurses.  Cplex: 10 min. Gap = 0.01%  
  Neos: 10 min. Gap (from Cplex) = 15.00%

- 4 weeks, 40 nurses.  Cplex: 10 min. Gap = 3%  
  Neos: 10 min. No feasible Sol'n.
Comparison of Models

- **Individual-Days model**
  - represents all feasible schedules (nearly!)
  - complex formulation is not user-maintainable
  - too long to solve for large instances.

- **Shift-Patterns model**
  - set of patterns ≠ set of all feasible schedules
  - constraints (mostly) enforced via patterns (data)
  - simpler formulation: end-user maintains data
  - solves faster (but still slow for large instances)
Rescheduling

- Circumstances change (nurse availability)
- So must reschedule, but do we
  - minimise the number of changes?
  - or recognise that different types of changes cause different amounts of disruption?
- Night shift on original day-off is more disruptive than Late → Early on same day
- How to measure disruption?
Disruption due to Rescheduling

- New 3rd objective: minimise disruption
  - Assign a penalty to each type of change
  - Minimise the sum of penalties
- Days model: penalise a shift change on a single day
- Patterns model: penalise a pattern change over a whole week
- Which is more realistic / easier to quantify?
Days Rescheduling Model

Data:

\( d_{sr} \): Disruption penalty of a change from shift \( s \) to shift \( r \) for a nurse.

\( a_{nsdw} \): If nurse \( n \) is assigned to shift \( s \) on day \( d \) of week \( w \) in the original schedule then \(-=1\), otherwise \(-=0\)

\( r_{dnw} \): If the change on day \( d \) of week \( w \) was requested (and previously agreed) by nurse \( n \) then \(-=1\), otherwise \(-=0\)

Min \( \sum_{s,d,w} \left( S^+_{sdw} + 0.001 S^-_{sdw} \right) + 0.1 \sum_{n,d,w,s,r} d_{sr} a_{nsdw} \left( 1 - r_{dnw} \right) x_{ndrw} \)
Patterns Rescheduling Model

Data:

\( d_{pq} \): disruption penalty of a change from pattern \( p \) to pattern \( q \) for a nurse.

\( a_{npw} \): If nurse \( n \) is assigned to pattern \( p \) in week \( w \) in the original schedule then \( = 1 \), otherwise \( = 0 \)

\( r_{nw} \): If the change in week \( w \) was requested (and previously agreed) by nurse \( n \) then \( = 1 \), otherwise \( = 0 \)

\[
\text{Min} \sum_{s,d,w} \left( S_{sdw}^+ + 0.001 S_{sdw}^- \right) + 0.1 \sum_{n,w,p,q} d_{pq} a_{npw} (1 - r_{nw}) x_{nrw}
\]
Rescheduling Issues

- Both models reoptimise in <1 second
- Lack of constraints to reschedule evenly
  - some nurses disrupted more than others
  → issues of fairness & model acceptance
- Mid-week rescheduling
  - Days model: can restart from any day in week 1
  - Patterns model: split week 1 from weeks 2, 3, ...
  → shorter patterns covering rest of week 1
Fairness between nurses

Example:
- Nurse 12 previously had week 4 off,
- now working two shifts (one is a night shift),
- is the only nurse to have any changes at all.
- Nurse 12 perceives the changes as unfair.

What to do?
- Negotiate a future week off, and fix in next run?
- More models constraints?
- Balance between over-modelling and human fix.
Conclusions so far

- Patterns model show (more) promise
- Model can consider complex (individual) nurse preferences when rescheduling
- Possible to develop a complete scheduling and rescheduling tool with
  - Freely available software (Neos) via www
  - User input via Microsoft/Open Office products
Future research 1

- Will often need to reschedule
  → detailed scheduling of later weeks is waste of effort if most of schedule never implemented

- Use model on a rolling horizon:
  - Detailed scheduling over (stable) short-term
  - Approximate scheduling after short-term (using smaller pattern set?)
Future research 2

- Include auto-generated files from Excel
- Generate new shift patterns
  - *Column generation*
  - Intelligent or human suggestions
- Develop both models to:
  - Consider individual preferences
  - Achieve “fairness” and other goals
- Validate & develop using real live data
  → seeking hospital to progress research