

Nurse Rescheduling

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Background

- Scheduling nursing staff is complex
- Need “good” schedules for nurses
- Should take account of nurses' preferences when scheduling
- ... and disruption when rescheduling
- But ...
- little research on nurse rescheduling



Literature

■ Nurse Scheduling

- Review (Personnel): Ernst et al (2004)
- Review (Nurses): Burke et al (2004)
- Preference-scheduling: Bard & Purnomo (2005ab)
- Self-scheduling: Baily et al (2007)
- Cyclic (fair but rigid): Bard & Purnomo (2007)
- Goal Progr (fair v. cover): Azaiez & Sharif (2005)
- Cyclic→Assign→Non-cyclic: Brucker et al (2005)
- Nice shifts ≠ more \$\$\$: Wright et al (2006)



Literature

- Nurse Rescheduling
 - Change schedule minimally. Optimal network ILP. If time > 10 min then heuristic: Moz & Pato (2003)
 - Faster formulation: Moz & Pato (2004)
 - Minimise dissimilarity & overtime. GA with heuristic: Moz & Pato (2007)
 - Reactive scheduling. Minimise differences & costs. IP + col gener'n: Bard & Purnomo (2005)
- Little research on how shift changes affect nurses.



Aim of Presentation

- Develop 2 nurse scheduling models
 - Individual days
 - Shift patterns
- Adapt models for rescheduling, so that
 - Quality is maximised
 - Disruption is minimised
- Consider nurses preferences and fairness of shifts changes when (re)scheduling



Typical Problem

- 20 permanent nursing staff
- Planning horizon of 28 days (4 weeks)
- 3 shifts/day: *Early, Late, Night*
- All $28 * 3 = 84$ shifts to be covered
- No floating nurses or on-call staff
- 3 contracts: *FullTime, PartTime, DayOnly*
- All nurses at same level and experience



Modelling & Solution Method

- Mixed Integer Programme (in AMPL)
- Model has multiple objectives
 - Main: minimise any nurse *shortages* on shifts
 - then: minimise *surplus* nurses on shifts
- Solvers:
 - Use Cplex v11.0, or Minto on *Neos* Server
 - Neos is free (via internet) and easy to use
 - Neos/Minto time limit of 10 min (self-imposed)



Messy Constraints

- Only 1 shift per day can be worked
- Cannot work *early* or *late*, after a *night*
- ≥ 2 days off per week
- ≤ 2 night shifts per week
- No more than FT/PT/DO contracted hours
- ≥ 1 weekend off in 4
- Avoid dispersed individual days off
- Work ≤ 5 days in a row



Individual Days Model 1

Data:

r_{sdw} Number of nurses required on shift s on day d in week w

h_s Number of hours in shift s

H_c Maximum contracted hours for contract c

$t(n)$ Type of contract of nurse n

Decision Variables:

x_{nsdw} Assigns nurse n to shift s on day d in week w ($= 1$, otherwise $= 0$)

S_{sdw}^+ Shortage of nurses on shift s of day d in week w , ≥ 0

S_{sdw}^- Surplus of nurses on shift s of day d in week w , ≥ 0



Individual Days Model 2

Minimise $\sum_s \sum_d \sum_w \left(S_{sdw}^+ - 0.001 S_{sdw}^- \right)$

such that

$$\sum_n x_{nsdw} + S_{sdw}^+ + S_{sdw}^- = r_{sdw} \quad \forall s, d, w$$

$$\sum_s x_{nsdw} = 1 \quad \forall n, d, w$$

$$\sum_s \sum_d x_{nsdw} \leq 5 \quad \forall n, w$$



Individual Days Model 3

$$\sum_d x_{n,\text{Night},d,w} \leq 2 \quad \forall n | t(n) \neq \text{DayOnly}, w$$

$$x_{n,\text{Night},d,w} = 0 \quad \forall n | t(n) = \text{DayOnly}, d, w$$

$$x_{n,\text{Night},d,w} + x_{n,\text{Early},d+1,w} + x_{n,\text{Late},d+1,w} = 1 \quad \forall n, d < 7, w$$

$$x_{n,\text{Night},7,w} + x_{n,\text{Early},1,w+1} + x_{n,\text{Late},1,w+1} = 1 \quad \forall n, w < 4$$

$$x_{n,\text{Late},d,w} + x_{n,\text{Early},d+1,w} \leq 1 \quad \forall n, d < 7, w$$

$$x_{n,\text{Late},7,w} + x_{n,\text{Early},1,w+1} \leq 1 \quad \forall n, w < 4$$

$$\sum_s \sum_d h_s x_{nsdw} \leq H_{c(n)} \quad \forall n, w$$



Individual-Days Model - Speed

- Normal capacity: 2 or 3 nurses per shift
 - Cplex: 4 secs to optimal zero solution
 - Neos: 10 minutes. Gap (from Cplex) = 100%
 - But uneven use of nurses.
- Tight capacity: 1 extra nurse per shift
 - Cplex: 10 minutes. Gap = 3%.
 - Neos: 10 minutes. No feasible solution found.



Shift-Patterns Model 1

Data:

r_{sdw} Number of nurses required on shift s on day d in week w

$c_{sdp} = 1$ if pattern p covers shift s on day d , otherwise 0

Decision Variables:

x_{npw} Assigns nurse n to shift pattern p in week w ($= 1$, otherwise $= 0$).

S_{sdw}^+ Shortage of nurses on shift s of day d in week w , ≥ 0 .

S_{sdw}^- Surplus of nurses on shift s of day d in week w , ≥ 0 .



Shift-Patterns Model 2

Minimise $\sum_s \sum_d \sum_w \left(S_{sdw}^+ + 0.001 S_{sdw}^- \right)$

such that

$$\sum_n \sum_p c_{sdp} x_{npw} + S_{sdw}^+ - S_{sdw}^- = r_{sdw} \quad \forall s, d, w$$

$$\sum_p x_{npw} = 1 \quad \forall n, w$$



Patterns model - speed

- Model chooses from just 26 possible 1-week shift patterns, so not exhaustively optimal.
- Normal capacity (2 or 3 nurses per shift)
 - Cplex: 0.23 secs to “optimal” (zero) solution
 - Neos: 1.62 secs to “optimal” (zero) solution
- Tight capacity (1 more nurse per shift)
 - Cplex: 33 secs to “optimal” (non-zero) solution.
 - Neos: 10 minute limit reached: Gap = 16.7%



Scaling up (tight capacity)

- 6 weeks, 20 nurses. Cplex: 229 secs. Optimal
Neos : 10 min. Gap (from Cplex) = 6.3%
- 4 weeks, 30 nurses. Cplex: 174 secs. Optimal
Neos: 10 min. Gap (from Cplex) = 7.6%
- 8 weeks, 20 nurses. Cplex: 10 min. Gap = 0.01%
Neos: 10 min. Gap (from Cplex) = 15.00%
- 4 weeks, 40 nurses. Cplex: 10 min. Gap = 3%
Neos: 10 min. No feasible Sol'n.



Comparison of Models

- Individual-Days model
 - represents all feasible schedules (nearly!)
 - complex formulation is not user-maintainable
 - too long to solve for large instances.
- Shift-Patterns model
 - set of patterns \neq set of all feasible schedules
 - constraints (mostly) enforced via patterns (data)
 - simpler formulation: end-user maintains data
 - solves faster (but still slow for large instances)



Rescheduling

- Circumstances change (nurse availability)
- So must reschedule, but do we
 - minimise the number of changes?
 - or recognise that different types of changes cause different amounts of disruption?
- Night shift on original day-off is more disruptive than Late → Early on same day
- How to measure disruption?.



Disruption due to Rescheduling

- New 3rd objective: minimise disruption
 - Assign a penalty to each type of change
 - Minimise the sum of penalties
- Days model: penalise a shift change on a single day
- Patterns model: penalise a pattern change over a whole week
- *Which is more realistic / easier to quantify?*



Days Rescheduling Model

Data:

d_{sr} : Disruption penalty of a change from shift s to shift r for a nurse.

a_{nsdw} : If nurse n is assigned to shift s on day d of week w in the original schedule then = 1, otherwise = 0

r_{dnw} : If the change on day d of week w was requested (and previously agreed) by nurse n then = 1, otherwise = 0

$$\text{Min } \sum_{s,d,w} \left(S_{sdw}^+ + 0.001 S_{sdw}^- \right) + 0.1 \sum_{n,d,w,s,r} d_{sr} a_{nsdw} (1 - r_{dnw}) x_{nr dw}$$



Patterns Rescheduling Model

Data:

d_{pq} : disruption penalty of a change from pattern p to pattern q for a nurse.

a_{npw} : If nurse n is assigned to pattern p in week w in the original schedule then = 1, otherwise = 0

r_{nw} : If the change in week w was requested (and previously agreed) by nurse n then = 1, otherwise = 0

$$\text{Min } \sum_{s,d,w} \left(S_{sdw}^+ + 0.001 S_{sdw}^- \right) + 0.1 \sum_{n,w,p,q} d_{pq} a_{npw} (1 - r_{nw}) x_{nrw}$$



Rescheduling Issues

- Both models reoptimise in <1 second
- Lack of constraints to reschedule evenly
 - some nurses disrupted more than others
→ issues of fairness & model acceptance
- Mid-week rescheduling
 - Days model: can restart from any day in week 1
 - Patterns model: split week 1 from weeks 2, 3, ...
→ shorter patterns covering rest of week 1



Fairness between nurses

- Example:
 - Nurse 12 previously had week 4 off,
 - now working two shifts (one is a night shift),
 - is the only nurse to have any changes at all.
 - Nurse 12 perceives the changes as unfair.
- What to do?
 - Negotiate a future week off, and fix in next run?
 - More models constraints?
 - Balance between over-modelling and human fix



Conclusions so far

- Patterns model show (more) promise
- Model can consider complex (individual) nurse preferences when rescheduling
- Possible to develop a complete scheduling and rescheduling tool with
 - Freely available software (Neos) via www
 - User input via Microsoft/Open Office products



Future research 1

- Will often need to reschedule
 - detailed scheduling of later weeks is waste of effort if most of schedule never implemented
- → Use model on a rolling horizon:
 - Detailed scheduling over (stable) short-term
 - Approximate scheduling after short-term (using smaller pattern set?)



Future research 2

- Include auto-generated files from Excel
- Generate new shift patterns
 - *Column generation*
 - Intelligent or human suggestions
- Develop both models to:
 - Consider individual preferences
 - Achieve “fairness” and other goals
- Validate & develop using real live data
→ seeking hospital to progress research